A Simple Experiment in Entanglement

It is possible to generate pairs of photos from a single source whose polarization is known to be identical. In quantum theory such photons are said to be *entangled*. In classical theory this concept is unknown; all that can be said about them is that the two photons have *identical polarization*.

In the experiment envisaged, such pairs of photons are directed towards two polarizers A and B and we shall be interested in the proportion of photon pairs which are either transmitted through *both* polarizers or absorbed by *both* polarizers.

Classical theory

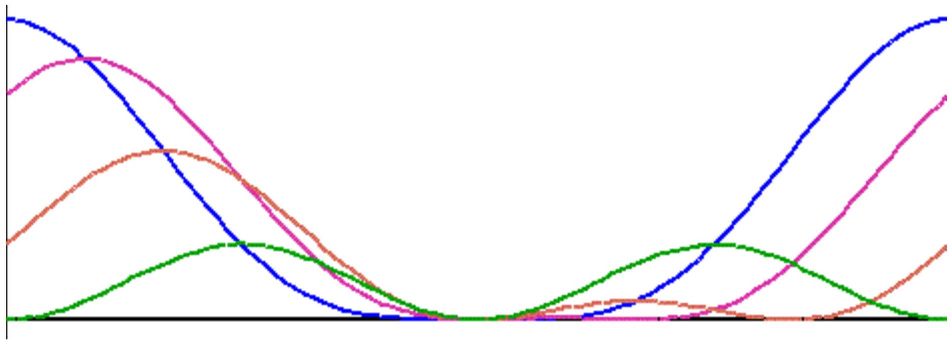
When a beam of polarized photons impinges on an ideal polarizer at an angle θ to the plane of polarization, the component of the electric vector which is transmitted is cos(θ) and the component which is absorbed is sin(θ). The proportion of energy which is transmitted is $\cos^2(\theta)$ and that absorbed in $\sin^2(\theta)$. Of course, $\cos^2(\theta) + \sin^2(\theta) = 1$ so energy is conserved.

We can therefore say that when an individual photon impinges on a polarizer at an angle θ, the *probability* of it being transmitted is $\cos^2(\theta)$ and the *probability* that it will be absorbed is $\sin^2(\theta)$

Now let us suppose that polarizer A is vertical and that polarizer B is inclined at an angle α to the vertical. If we assume the principle of separability (ie that no influence can be transmitted from A to B or vice versa) then the probabilities are independent and therefore the probability that both photons will be transmitted is the product of the probabilities that each will be transmitted. Hence, for a pair of photons polarized at an angle of θ to the vertical, the probability *p* that they will be transmitted through both polarizers is:

$$p = \cos^2(\theta) \,.\, \cos^2(\theta - \alpha) \tag{1}$$

This function looks like this (with θ running from 0 to π) with α set to 0º (blue), 30º (purple), 60º (brown) and 90º (green).

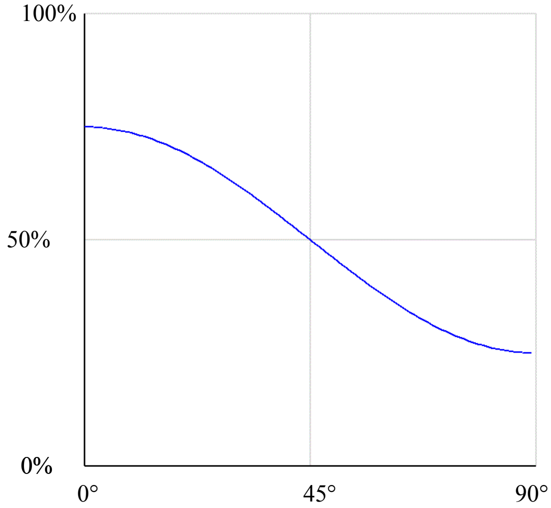


If we experiment with a large number of photon pairs with random polarization, the proportion of photons *P* which are transmitted through both polarizers will be:

$$P = \frac{1}{\pi} \int_0^{\pi} \cos^2(\theta) \,.\, \cos^2(\theta - \alpha) \; d\theta \tag{2}$$

which is $$P = \frac{1}{4}\left(1+\frac{\cos 2\alpha}{2}\right) \tag{3}$$

Since the proportion absorbed by both polarizers will be the same, the proportion of photons which is either absorbed of transmitted by both polarizers will be double this. This function looks like this:



When the two polarizers are parallel, only 75% of the photons do the same thing because those pairs of photons which are polarized at angles other than vertical or horizontal have a finite chance of doing something different.

Quantum theory

In the quantum world, the situation is different. Let us suppose that the vertical polarizer (A) is slightly closer to the source than polarizer B[1]. The observation of the photon at A confirms that the photon reaching A is either vertically polarized or horizontally polarized – and that, *because the two photons are entangled*, the other photon is also similarly polarized. Under a conventional interpretation you can say that the observation of A collapses the wave function and creates a photon approaching B which is either vertically or horizontally polarized. Note carefully that the observation of the photon being either transmitted or absorbed at A *excludes* the possibility that the photon was polarized at any other angle.

Now, by symmetry, half the (randomly polarized) photons reaching A will be transmitted and half will be absorbed. For those photons which are transmitted, we know that its counterpart is vertically polarised and when it reaches polarizer B which is inclined at an angle α, the probability that it will also be transmitted is, of course, $\cos^2(\alpha)$. So out of a large number of photons, the proportion passing through both polarizers will be

$$P = 1/2\cos^2(\alpha) \tag{4}$$

Similarly, the proportion of photons which are both absorbed will be exactly the same.

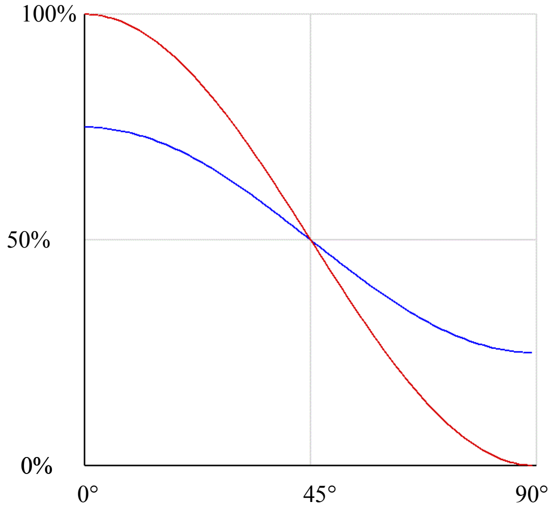
It follows that the proportion of photons which are either transmitted or absorbed will be:

$$P = \cos^2(\alpha) \tag{5}$$

This makes sense because wharn the polarizers are parallel (i.e. $\alpha = 0$) then if the photon passes through A, we know that the other photon must pass through B. And if the first photon is blocked at A, the second photon must be blocked at B.

1 Actually this condition is irrelevent but it serves to make things a bit easier to talk about.

The differences are best shown on a graph with the classical prediction in blue and the quantum prediction in red:



The difference is dramatic. Quantum theory predicts a correlation between the behaviour of the two photons which cannot be explained by the classical assumption that the two photons are simply polarised in parallel. It seems that the passage of the first photon through polarizer A physically causes the second photon to adopt the same polarization. Einstein was horrified at this and called it 'spooky action at a distance'.

A 'hidden variable' theory

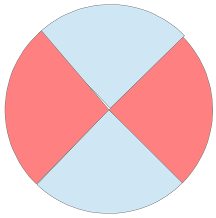
Einstein's conclusion was that quantum theory was incomplete and that, unknown to us, there were 'hidden variable' which accounted for the correlations. But in 1964 an Irish physicist named John Stewart Bell proved a theorem which implied that no theory using *local hidden variables* could ever reproduce the correlations observed. We shall try to construct a local hidden variable theory to explain the behaviour of the two entangled photons.

What we mean by a local hidden variable is a potentially measurable and unchanging quantity such as mass or frequency which an entity like a photon carries with it from the moment the entity is created to the moment it is destroyed. Polarization is such a quantity but, as we have seen, just insisting that the two photon start out life with parallel polarization does not produce the desired correlations.

So let us suppose that, instead of a fixed plane of polarization, the two photons decide, in advance as it were, on certain angles of polarization which they will pass though and those that they will not pass through. They could do this in an infinite number of ways, but provided they agree on the same angles, that will ensure that when they encounter polarizers at the same angle, they will do the same thing (i.e. be transmitted or absorbed).

Now in order to ensure that when they encounter polarizers at right angles, they will always do different things, they must simply pair up their 'good' and 'bad' angles in complementary pairs.

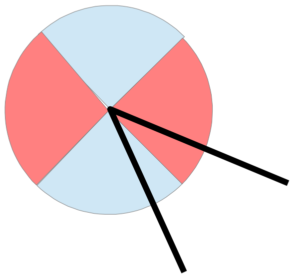
Again, there are still an infinite number of ways they could do this but the simplest way is to divide up all the angles into four quadrants as in the following diagram where the blue angles are those which will transmit the photon and the red angles are those which will absorb the photon.



The only problem with this theory is that if the polarizers are vertical, both photons will always be transmitted – but in reality, half will be transmitted and half will be absorbed; Quantum theory only insists that they will *do the same thing*.

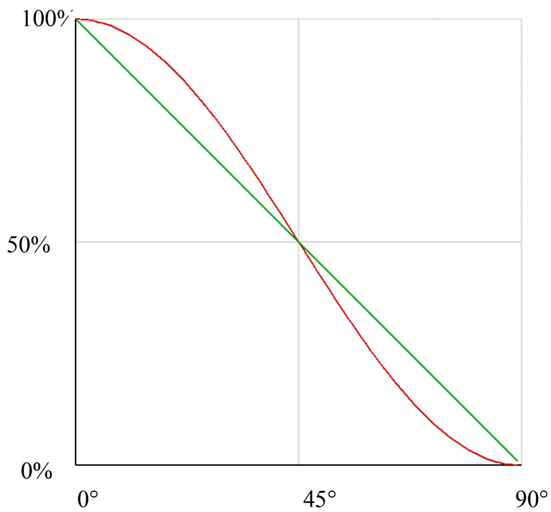
This objection is easily met by supposing that the two photons spin the disc to a random angle before setting off. Now both conditions will be met. Parallel polarizers will result in 100% correlation; polarizers at right angles will result in 100% anti-correlation.

What about polarizers set at 45°? I think it is pretty obvious that there is a 50:50 chance that the angles of the two polarizers will lie in the same quadrant and this agrees perfectly with the prediction of quantum theory because $\cos^2(45°) = ½$.



But what if the polarizers are set to some arbitrary angle θ? Suppose we have two pointers which we can move around the disc. What is the probability that both pointers will lie in the same quadrant? We have already determined that if the angle between the pointers is 0°, they must both be in the same quadrant; and if the angle is 90°, they must be in different quadrants. As you increase the angle from 0° to 90° the probability must decrease in *linear proportion*. For example, if the pointers are 30° apart, then out of the 12 ways in which you can line up the pointers on the disc, 8 will lie in the same quadrant and 4 in different quadrants. The probability of lying in the same quadrant is therefore ⅔.

But $\cos^2(30°) = ¾$, not ⅔. The difference between the quantum prediction (in red) and the hidden variable prediction (in green) is best shown on a graph.



The differences may be slight but they are of enormous significance. The green line represents the *best possible correlation* on the basis of a local hidden variable theory – but when the polarizers are set to angles between 0° and 45°, the hidden variable theory underestimates the actual results by up to 14%; and when the polarizers are set to angles between 45° and 90°, the hidden variable theory overestimates the actual results by the same amount.

Conclusions

What are we to make of this?

Well, firstly, Einstein's dream of a local hidden variable theory is finished. Bell's theorem and many subsequent experiments have killed that goose.

So do we have to accept 'spooky action at a distance' after all?

Not necessarily. It is often said that Bell's theorem forces us to choose between locality (i.e. the assumption that events in one place cannot instantaneously affect events at another) and realism (the idea that entities such as photons have permanent, potentially measurable quantities at all times.

I am not so sure. I agree that we must change something and that of the two, I think it is our concept of reality which must change. But I do not believe that we just have to abandon reality altogether and accept that objects like photons do not have measurable properties until they are measured. That is like saying that the Moon does not exist until we look at it (another of Einstein's objections to quantum theory!).

I have an idea that, for a while, multiple realities can exist at the same time. In particular, when the entangled photons leave the source, there are an infinite number of possibilities for their polarization – but *Nature has not yet decided which* is going to count as real. When one of the photons reaches a polarizer, its plane of polarization becomes fixed. All the other possibilities disappear instantly leaving the second photon with the same polarization as the first.

If you think this sounds a bit like 'spooky action at a distance', consider this analogy: suppose you enter a raffle with 100 tickets for sale at £1 each with a prize of £100. Before the draw, the raffle tickets are worth £1 each and could even be used as currency. But as soon as the winning number is revealed, the ticket bearing that number is worth £100 and the others are instantly rendered worthless. There is no need here for any instantaneous communication between the tickets so there is no violation of locality.

We do, however, have to accept that reality is not quite what we though it was. Reality can be suspended for a while and in general it can only be determined in retrospect.

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